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Moment Magnitude–Local Magnitude Calibration for Earthquakes in Western Canada

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Abstract Local magnitude (M_L) is the primary magnitude scale calculated for western Canada by the Geological Survey of Canada (GSC). Moment magnitude (M_w), derived from moment tensor analysis, provides a more robust estimate of the magnitude of earthquakes but is more demanding to calculate. Moment tensor analysis of regional seismic data for earthquakes with magnitudes larger than 3.5 in western Canada is now possible owing to the installation of more than 40 three-component broadband stations in western Canada, the Pacific Northwest of the United States, and southeast Alaska. More than 100 regional moment tensor solutions have been calculated in the Canadian Cordillera and Vancouver Island/Puget Sound region for 1996–2004 at the GSC. These solutions, along with 45 prior solutions, allow the calibration of M_w–M_L throughout much of western Canada. Continental crust events in the Canadian Cordillera and Vancouver Island/Puget Sound region are found to have $M_w = M_L$ for earthquakes with $M_L \geq 3.6$. In contrast, earthquakes located within the subducting slab in the Vancouver Island/Puget Sound region, where there are complex source–receiver travel paths, have M_w systematically larger than M_L by nearly 0.6 magnitude units. The calibrations of M_w with M_L are an important result that will allow the western Canadian earthquake database to be used more effectively for tectonic studies and seismic hazard analysis.

Introduction

Local magnitude (M_L) was the first magnitude scale to be developed in an attempt to quantitatively describe the size of an earthquake. It was developed by Richter (1935) specifically for earthquakes in southern California. M_L is now widely used, often in tectonic environments that can be completely different from southern California. Therefore, it is necessary to calibrate M_L so that it gives magnitudes that are consistent with other magnitude scales such as body-wave magnitude (m_b) or moment magnitude (M_w).

With the recent ability to calculate regional moment tensor (RMT) solutions for earthquakes as small as $M \sim 3.5$ in western Canada, it has become possible to build a large catalogue of M_w values for earthquakes in the Canadian Cordillera and the Vancouver Island/Puget Sound region. At the Geological Survey of Canada (GSC), 102 RMT solutions for events in this region have been calculated for 1996–2004 with M_w ranging from 3.4 to 6.8. These solutions, along with 30 RMT solutions calculated by Oregon State University (OSU) in 1994–1995 (Braunmiller and Nábělek, 2002) and 15 Harvard centroid moment tensor (CMT) solutions from 1976–1993 (Harvard CMT catalog), allow for a M_w/M_L calibration for earthquakes in the Canadian Cordillera and Vancouver Island/Puget Sound region. M_w is the preferred magnitude for seismic hazard analysis, and determining how other magnitude scales are related to it is important, as M_w estimates are not always available. The purpose of this study was to determine how M_L relates to M_o (seismic moment) and M_w in the Canadian Cordillera and Vancouver Island/Puget Sound region. This article complements that of Ristau *et al.* (2003), which determined the M_w/M_L relationship for the offshore region of western Canada, and completes the M_w/M_L calibration for the most seismically active regions of western Canada.

The Magnitude Scale

A brief summary of M_L and M_w are given here; Ristau *et al.* (2003) provide a more detailed description. Richter (1935) defined M_L as

$$M_L = \log A - \log A_0 \tag{1}$$

where A is the amplitude of the horizontal ground displacements of the earthquake and A_0 is that of a reference event, at a distance of 100 km, as measured on a Wood–Anderson seismograph. In western Canada, the method used to calculate M_L is similar to that of Richter (1935); it uses the original $\log A_0$ values and applys the calculations in the distance range of 50–600 km. Stations with an epicentral distance of less than 50 km are not used to calculate M_L in order to minimize the effect of focal depth. The main difference is that the maximum amplitude used for earthquakes in western Canada has been consistently calculated from the vertical component since routine magnitude calculation began in 1955, whereas Richter (1935) used the horizontal amplitudes from Wood–Anderson seismographs. There is also a period correction to adjust for the different period responses of the seismographs used in western Canada compared with a Wood–Anderson seismograph.

Magnitude estimates (e.g., M_L, m_b) are limited by making frequency-dependent measurements of the amplitudes of surface or body waves, which results in magnitude saturation for larger earthquakes. M_w, first proposed by Kanamori (1977), is computed directly from M_o, which is not dependent on the frequency so long as the frequencies used are well below the corner frequency. M_o does not saturate and is a fundamental physical quantity directly related to the earthquake fault area and the average slip on the fault. It is the best representation of the size of earthquake, and M_w gives a magnitude estimate directly related to M_o. Hanks and Kanamori (1979) derived a relationship for calculating M_w with respect to M_o that is consistent with the M_L–M_s (surface-wave magnitude scale) relation. It is given by

$$M_w = \tfrac{2}{3} \log M_o - 10.7 \qquad (2)$$

where M_o is in dyne cm. Determination of M_o is much more computationally extensive than magnitude calculation; however, several program packages have been developed to make the process routine, such as the code used in this study (Ammon, 2001). Therefore, how M_L relates to M_o can be determined by finding out how M_L relates to M_w.

Regional Moment Tensor Analysis

Since the mid-1990s more than 40 three-component broadband seismometers have been installed in western Canada, the Pacific Northwest of the United States, and southeast Alaska, and they now provide high-quality seismic data suitable for moment tensor analysis (Fig. 1). These data, along with improved methods and increased computing power, have made it possible to routinely calculate RMT solutions for smaller earthquakes in western Canada, that is, $M_w \geq\sim 3.5$. Regional moment tensor solutions differ from Harvard or USGS moment tensor solutions in that they use regional data (source–receiver distances of ~1000 km or less) and region-specific Earth models. For $M \leq\sim 5.0$ the signal-to-noise ratio at the low frequencies used in global analysis is poor, and higher frequencies must be used. Higher frequency signals are more complicated, and the generic whole-Earth models used in global moment tensor analysis are not sufficiently detailed to accurately model the observed waveforms. Earth models specific to the source region must be used in order to adequately model the observed data. By extending the magnitude threshold down to M_w ~3.5, it has been possible to calculate more than 10 times as many moment tensor solutions compared with teleseismic methods. Because of low signal-to-noise ratios at low frequencies (below the corner frequency) and the inability to properly model higher frequency signals, it is not always possible to calculate a RMT solution for earthquakes with M_w <4.0; therefore, the catalog for events with M_w <4.0 is not complete.

The RMT solutions presented here were calculated using the code of Ammon (2001) as developed for use on smaller magnitude earthquakes. For the majority of the events, the observed waveforms are bandpass filtered between 20 and 50 sec (0.02-0.05 Hz), which generally provides a good signal-to-noise ratio for modeling the observed waveforms (e.g., Braunmiller and Nábêlek, 2002). In this frequency range the seismograms are dominated by guided waves and surface waves, which can be adequately modeled using 1D velocity models. Harvard CMT solutions typically use waveform energy with periods greater than 45 sec (less than 0.022 Hz) (Dziewonski *et al.*, 1981), and USGS moment tensor solutions use a 15–55 sec (0.018–0.067 Hz) passband (Sipkin, 1986). Details on the moment tensor algorithm and method can be found in Ammon (2001) and Ristau (2004) and will not be discussed here.

Moment Magnitude–Local Magnitude Calibration

This article is the first study to calibrate M_w with M_L for all of the most seismically active onshore regions of western Canada. It incorporates the results of Hyndman *et al.* (2005) for the northern Cordillera, and the work of Bolton (2003), who compared M_w with M_L for seven in-slab events in the Vancouver Island/Puget Sound region. Ristau *et al.* (2003) compared M_w with M_L for more than 260 earthquakes off the west coast of Vancouver Island and in the Queen Charlotte Islands region. The travel paths of those earthquakes contain substantial segments of oceanic lithosphere, which have very different wave transmission characteristics compared with continental crust. The essential point is that the L_g phase, which is normally used to calculate M_L, does not propagate through oceanic crust when the length of the oceanic crustal path is greater than 100–200 km owing to the effect of the thin crust on the crustal wave guide (Press and Ewing, 1952; Zhang and Lay, 1995). Therefore, M_w–M_L relations for the offshore region cannot be extrapolated to the onshore regions. Ristau *et al.* (2003) contains a detailed discussion on the problems of calculating M_L for earthquakes occurring off Canada's west coast.

Nearly 150 moment tensor solutions have been calculated for the Canadian Cordillera and the Vancouver Island/Puget Sound region by the GSC, OSU, and Harvard, ranging from M_w 3.4–7.5 (Fig. 2). To study the M_w/M_L relationship, all of the events occurring in the continental crust are grouped together regardless of whether they are located in

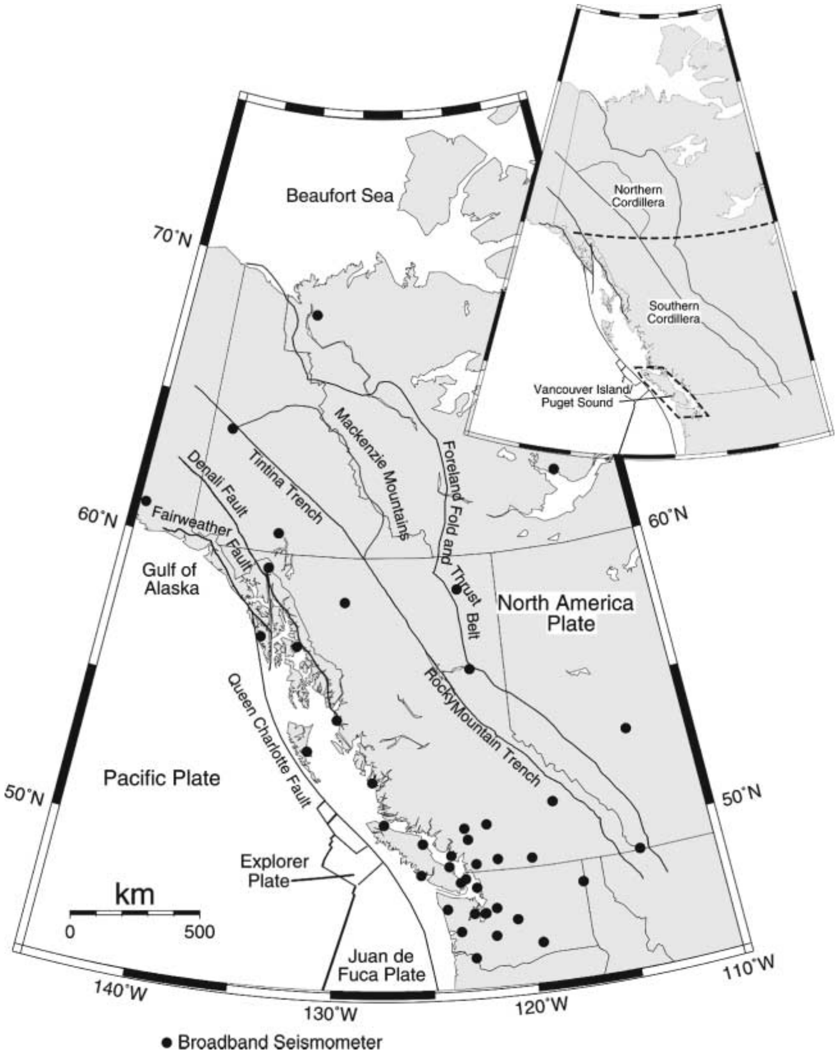

Figure 1. Locations of three-component broadband seismometers in western Canada, the Pacific Northwest of the United States, and southeast Alaska through to the beginning of 2005. These stations are routinely used to calculated RMT solutions in western Canada and adjacent regions. Also shown are the major tectonic features of western Canada. Inset: locations of the Canadian Cordillera and the Vancouver Island/Puget Sound regions.

the Canadian Cordillera or the Vancouver Island/Puget Sound region. Then, events located in just the Vancouver Island/Puget Sound region are considered to study the M_w/M_L discrepancy for events occurring within the overlying North America crust compared with those occurring within the subducting Juan de Fuca slab.

Continental Crust Events

A total of 131 continental crust events with M_w ranging from 3.4 to 7.5 are used to compare M_w with M_L. It has been shown that a unique moment tensor solution can often be obtained using as few as one or two three-component broadband stations (e.g., Dreger and Helmberger, 1993; Delouis and Legrand, 1999). However, inaccuracies in modeling waveforms using 1D velocity models, particularly when the source–receiver distances are several hundred kilometers, can result in nonunique and unstable solutions. Moment tensor solutions calculated using only one or two stations can depend greatly on the velocity model used to calculate the Green's functions. Solutions calculated using several stations are generally more reliable and more stable (e.g., Dreger, 2000). In this study of the Canadian Cordillera, we typically use 5–10 stations having a good azimuthal distribution with source–receiver distances that are often greater than 500 km.

Figure 3 (top) compares $\log(M_o)$ with M_L for all events occurring within the North America plate, with the best-fit line (thin solid line) and 95% confidence limits (dashed lines) also shown. The thick black line is the Hanks and Kanamori (1979) relationship (equation 2), solved for M_o and replacing M_w with M_L, which represents a 1:1 relationship between M_w and M_L. The Hanks and Kanamori (1979) relationship falls outside of the 95% confidence limits for most of the data set, suggesting that M_w and M_L are not consistent when using the entire data set. In Figure 3 (top), events with M_L ~3–4 have M_o values consistently on the high side of the Hanks and Kanamori (1979) line. Continental crust earthquakes with $M_L \leq \sim 3.5$ are at the lower magnitude limit for being able to calculate RMT solutions and generally have a worse fit between the synthetic and observed waveforms than events with $M_L > 3.5$. Therefore, the

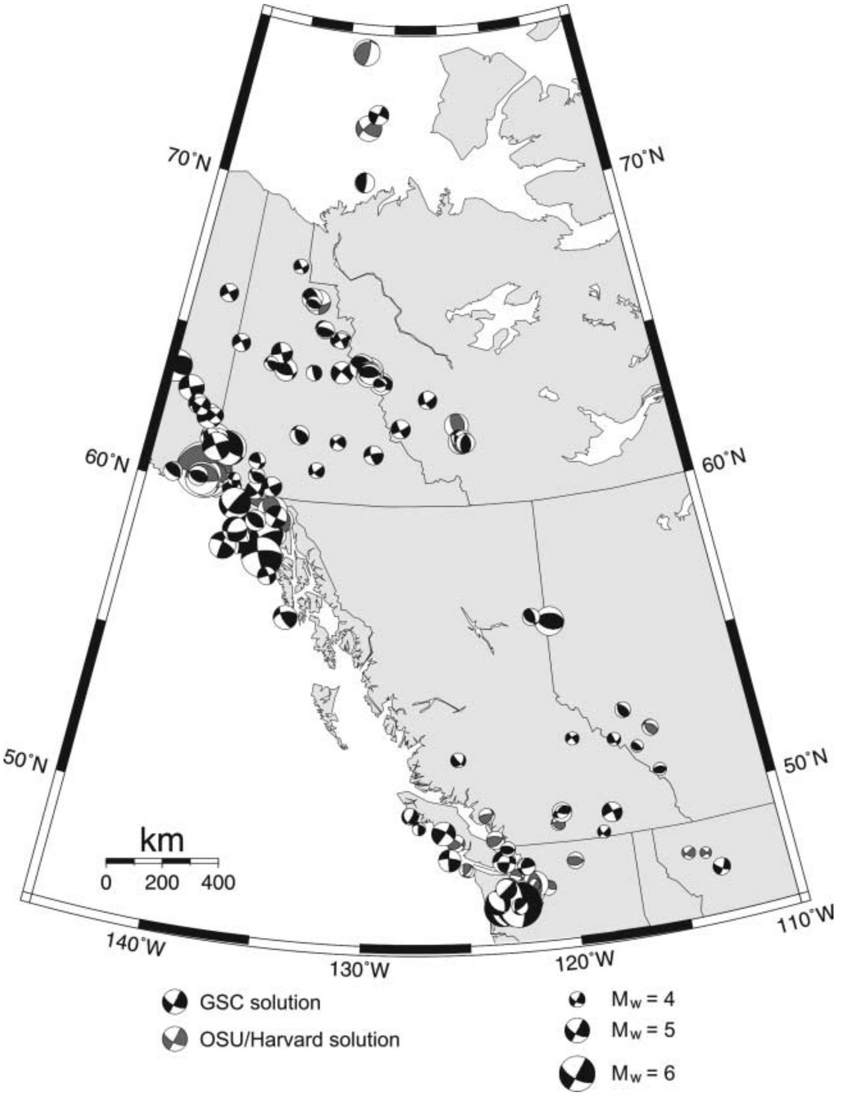

Figure 2. Moment tensor solutions in the Canadian Cordillera and Vancouver Island/Puget Sound region calculated by the GSC, OSU, and Harvard shown as lower hemisphere fault mechanisms.

M_o values may not be reliable. To determine the lowest M_L at which M_L and M_o are consistent with the Hanks and Kanamori (1979) relationship, the data set is cut off at varying M_L levels. Figure 3 (middle) and (bottom) show the results using a minium M_L of 3.4 and 3.6 respectively. For $M_L \geq 3.6$, there is excellent agreement at the 95% confidence limits between the data presented in this study and the Hanks and Kanamori (1979) relation.

Overlying Crust Versus Subducting Slab

There are 38 events available for the Vancouver Island/Puget Sound region—23 located in the overlying crust of the North America plate, 13 in the subducting slab of the Juan de Fuca plate, and two that may be either overlying crust or subducting slab events. Figure 4 (top left) compares M_w with M_L for all available events in the Vancouver Island/Puget Sound region regardless of magnitude, using the method of Ristau *et al.* (2003). The results do not vary significantly whether the uncertain events are considered as overlying crust or subducting slab events; therefore, we have considered those events as in-slab events. The dashed line represents an ideal 1:1 relationship between M_w and M_L, and the solid line is a best-fit line assuming a slope parallel to the 1:1 line. The offset is calculated by taking an average of the residuals between M_w and the 1:1 line, and the standard deviation of the residuals is used as the error. The overlying crust events are mainly smaller magnitude events and have M_w 0.12 $\pm$ 0.29 magnitude units larger than M_L.

There is no method to quantitatively calculate the uncertainty on the seismic moment by way of moment tensor solutions. Ekström and England (1989) estimated an error in the seismic moment for Harvard CMT solutions by comparison of CMT results with moment estimates from geodetic and other seismic data. They estimated an error of 20% for the moment, which corresponds to a factor of 1.4 uncertainty at the 95% confidence level. In a tectonically complex region such as the Vancouver Island/Puget Sound region, it is difficult to accurately model the observed waveforms using a 1D Earth model. Therefore, we use a conservative estimate

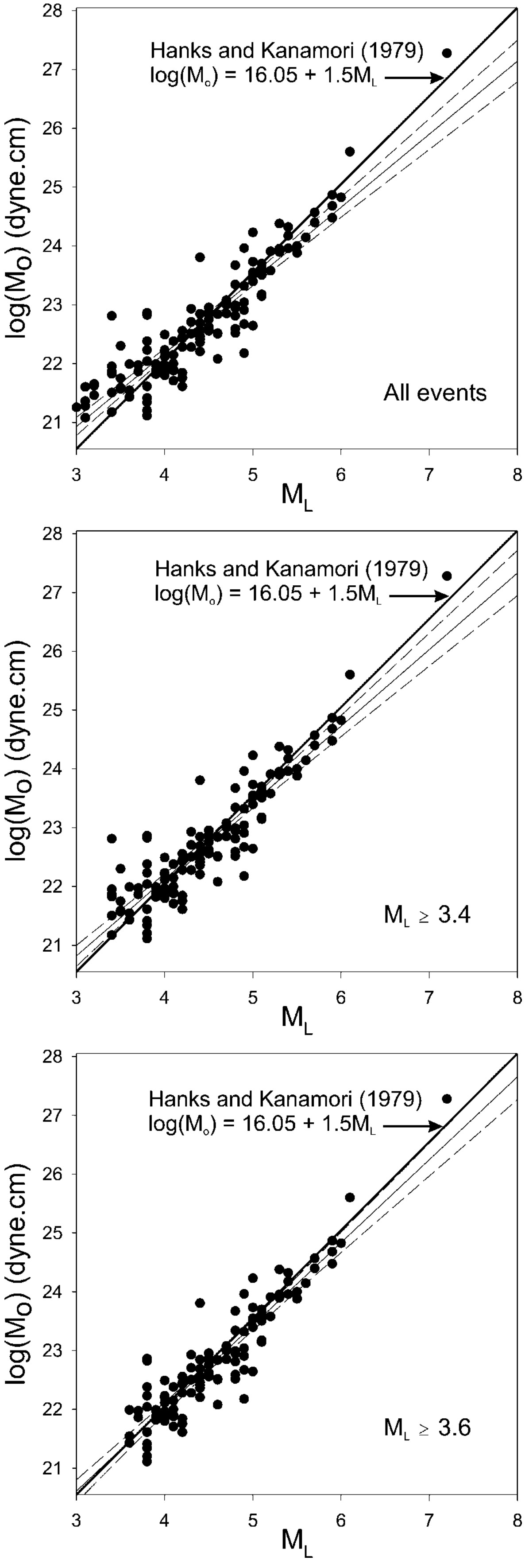

Figure 3. M_o/M_L relationship for continental crust earthquakes in the Canadian Cordillera and Vancouver Island/Puget Sound regions. The thin solid line is a best-fit line between log (M_o) and M_L, and the dashed lines are 95% confidence limits. The thick solid line is the Hanks and Kanamori (1979) relation with M_L in place of M_w. (Top) All continental crust events; (middle) events with $M_L \geq 3.4$; (bottom) events with $M_L \geq 3.6$. When M_L is ≥ 3.6 the Hanks and Kanamori (1979) relation falls within the 95% confidence limits, which shows that M_w and M_L have a 1:1 relationship for $M_L \geq 3.6$.

on the uncertainty in the seismic moment of a factor of 2, which corresponds to approximately a 0.2 uncertainty in magnitude. Considering a M_w uncertainty of 0.2, the M_w/M_L discrepancy for events occurring in the overlying crust is likely not significant statistically. Similar to the data set in Figure 3, events with $M_L \leq 3.5$ in Figure 4 (top left) have M_w values consistently higher than M_L. When comparing only overlying crust events with $M_L \geq 3.6$ as in Figure 3 (bottom), M_w and M_L are equivalent (Fig. 4, top right).

The in-slab data set (Fig. 4, bottom) is small but with a wide range of magnitudes. The subducting slab events have M_w almost 0.6 magnitude units larger than M_L, which is significant even considering a factor of 2 uncertainty in calculating M_o. The 0.6 magnitude discrepancy is observed regardless of whether the entire data set is used (Fig. 4, bottom left) or only events with $M_L \geq 3.6$ (Fig. 4, bottom right), demonstrating that in-slab events need to be considered separately from overlying crust events. The larger discrepancy between M_w and M_L for in-slab events is likely related to the more complex source–receiver travel path, which can have a significant effect on M_L estimates.

Discussion and Conclusions

We have determined that, on average, M_w is equivalent to M_L with an uncertainty of about 0.2 magnitude units for earthquakes with $M_L \geq 3.6$ occurring in the continental crust of the Canadian Cordillera and the Vancouver Island/Puget Sound region. Earthquakes with M_L of ~3.6 are at about the lower limit for being able to calculate RMT solutions, and M_o values for smaller earthquakes may not be reliable. The 1:1 relationship between M_w and M_L also holds when considering overlying crust events located only in the Vancouver Island/Puget Sound region. Events located in the subducting slab in the Vancouver Island/Puget Sound region have M_w nearly 0.6 magnitude units larger than M_L. This is a critical new result, as large, damaging earthquakes can occur within the subducting slab and accurate magnitude and, particularly, seismic moment estimates are important for seismic hazard analyses and tectonic studies (e.g., Mazzotti *et al.*, 2002; Hyndman *et al.*, 2003). The magnitude relationship between crustal and slab events is an issue that can be addressed in future studies using more detailed Earth

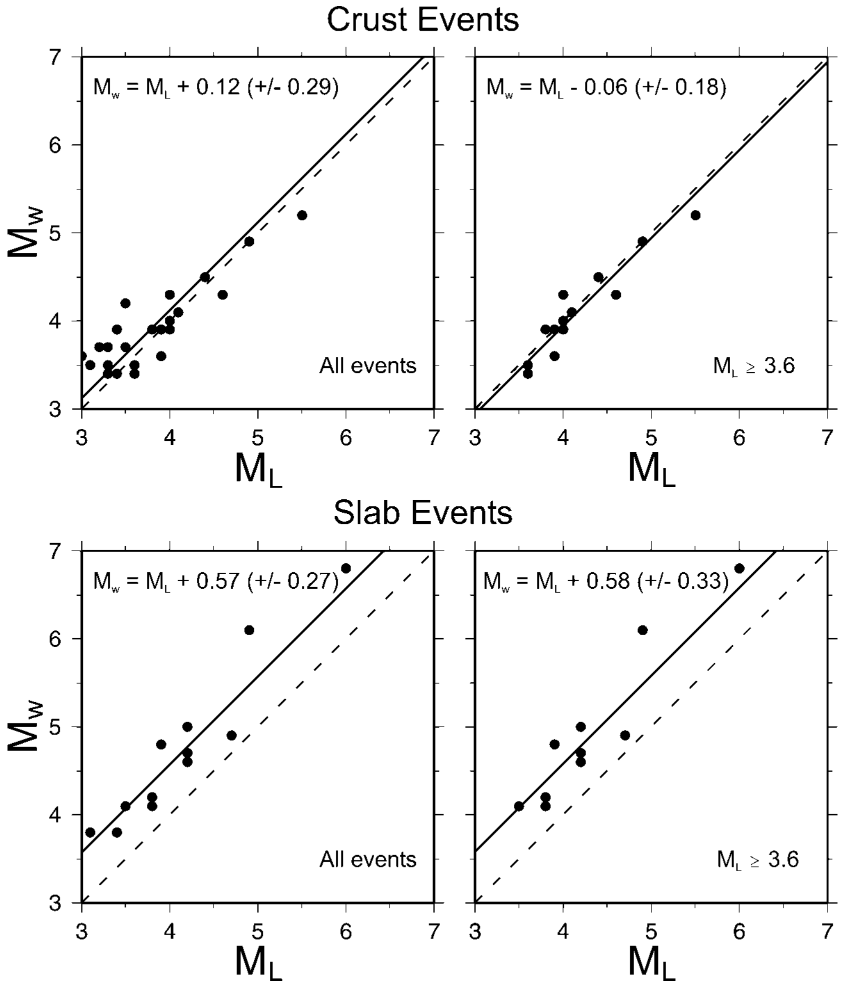

Figure 4. M_w/M_L comparison for events in the Vancouver Island/Puget Sound region. In each case the dashed line represents a 1:1 relationship between M_w and M_L and the solid line is a best-fit line assuming a slope parallel to the 1:1 line. The M_L correction is indicated for each region. (Top) Earthquakes occurring within the overlying crust; (bottom) earthquakes located within the subducting slab.

models and carefully examining the effect of the subducting slab and overlying crust on modeled waveforms. Establishing that M_w and M_L have a 1:1 relationship for continental crust events in the Canadian Cordillera and Vancouver Island/Puget Sound region is important as M_L is the primary magnitude calculated by the GSC for earthquakes in western Canada and has been calculated in a similar manner since 1955. M_L values in the western Canadian earthquake database can now be converted to M_w and M_o, which can then be used for seismic hazard analysis and tectonic studies. This study completes the M_w/M_L calibration for the seismically active onshore and offshore regions of western Canada.

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